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Mechanism of Heat Transfer to Liquid Drops

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The purpose of this investigation was to study the process of heat transfer to a liquid drop rising through another liquid. In experimental runs various-sized drops of S.A.E.-10 lubricating oil, kerosene, and xylene were heated with water; also, water drops were heated with various organic liquids. Theoretical calculations of temperature change were made, various mechanisms of heat transfer being assumed. The temperature change predicted with each mechanism was compared with the experimental results and the controlling mechanism thus determined for each system studied.

A number of chemical engineering operations involve the transfer of heat and mass between a continuous fluid phase and a second phase such as a gas bubble, liquid drop, or solid sphere moving relative to the continuous phase. A thorough understanding of all the factors which govern this type of process has not been attained; however, a considerable amount of work has been done in recent years on segments of the over-all problem, and the present work on heat transfer to liquid drops is an attempt to fill in another portion of the picture.

No information was found in the literature concerning experimental measurements of heat transfer rates to liquid drops from other liquids. In the field of mass transfer, however, several investigators have studied the mechanism of solute extraction from drops (6 and 14). The apparatus used for these experiments was similar to the equipment employed in the present study of heat transfer. In the mass transfer apparatus a solute was extracted from the drops as they moved

through a continuous liquid phase. The average concentration of the solute in the drops was measured before and after the extraction period. This procedure is analogous to measuring the heat transferred to a drop from a surrounding liquid by determining the average inlet and outlet drop temperatures.

Several factors could affect the heat transfer rate to a drop. A film of liquid surrounding the moving drop could cause appreciable resistance to heat transfer. If the liquid in the drop were stagnant, radial conduction would control the rate of heat transfer beneath the drop surface, but if there were circulation within the drop, the heat transfer rate would increase; consequently, the outside film coefficient, the thermal diffusivity of the liquid in the drop, and the extent of internal circulation appear to be the most important factors in the heat transfer process.

Kramers (11) developed an empirical equation for the film coefficient of a fluid flowing past steel spheres heated by high-frequency induction and maintained at a constant, uniform temperature. He used

various sphere diameters from 0.70 to 1.27 cm. and correlated the experimental data as follows:

$$Nu = 2.0 + 1.3(Pr)^{0.15} + 0.66(Pr)^{0.31}(Re)^{0.50} \quad (1)$$

Film coefficients calculated from this equation are probably applicable to heat transfer between continuous fluid and a stagnant drop. When the equation was used to calculate an outside water-film coefficient, values were obtained between 550 and 800 B.t.u./(hr.)(sq. ft.)(°F.). Organic liquid-film coefficients were between 130 and 170 B.t.u./(hr.)(sq. ft.)(°F.). However, in the case of a circulating drop such a coefficient would serve only as an approximation.

Equation (2) has been developed for calculating the temperature change of a sphere being heated by the mechanism of pure radial conduction (10). In the development of this equation it was assumed that there was no outside film resistance and that the temperature of the drop surface was the same as the average

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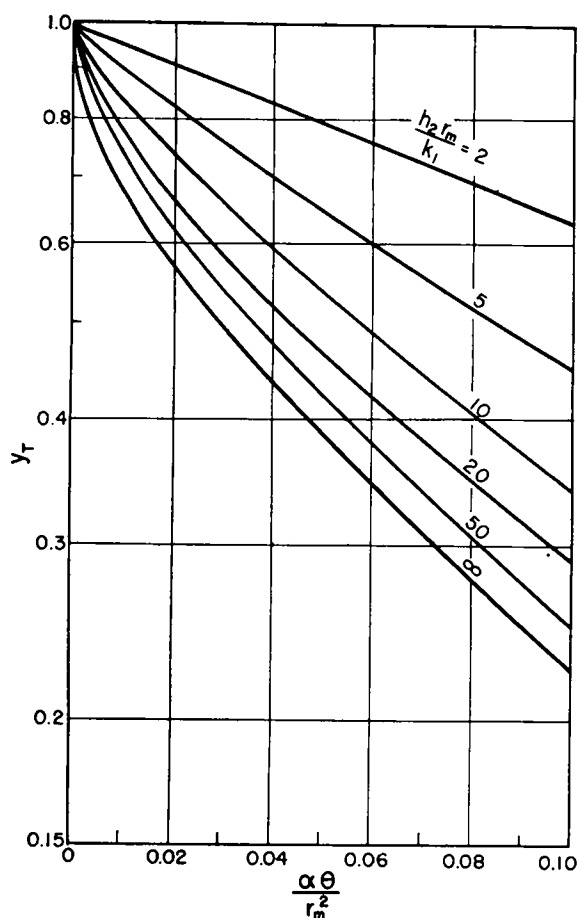


Fig. 1. Temperature change of a noncirculating drop with varying outside resistance.

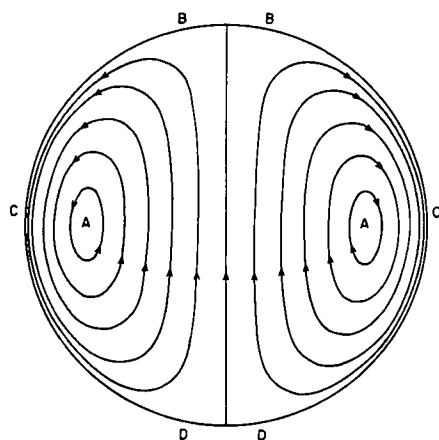


Fig. 2. Circulation pattern for rising drop.

temperature of the surrounding medium.

$$Y_T = \frac{6}{\pi^2} \left[\exp \frac{-\alpha \pi^2 \theta}{r_m^2} + \frac{1}{4} \exp \frac{-4\alpha \pi^2 \theta}{r_m^2} + \dots \right] \quad (2)$$

Jakob (10) gives an equation for the case in which the outside film coefficient becomes small enough so that the outside resistance is an appreciable part of the over-all resistance to heat transfer. Charts

are given in his book from which the temperature rise of a noncirculating drop with an outside film resistance can be predicted. A modified version of these charts is reproduced in Figure 1.

If a drop circulates internally, the heat transfer rate may be greatly increased. The hydrodynamics of falling drops undergoing viscous circulation was investigated by Hadamard (7). Using his results Kronig and Brink (12) developed a mathematical expression for the amount of solvent extracted from a drop which has internal circulation and negligible outside film resistance to mass transfer. The circulation pattern for a rising drop is shown by the streamlines in Figure 2. The liquid near the outside of the drop flows downward as a result of the drag forces exerted by the surrounding liquid. In order to calculate the amount of diffusion that would occur, Kronig and Brink assumed that the concentration of solvent in the drop was the same at any position on a given streamline. The analogous assumption for the heat transfer case is that all the streamlines are isotherms. If each streamline is to be at the same temperature throughout its length, the time required for the liquid to circulate must be small compared with the time required for the drop to cool. The critical ratio of

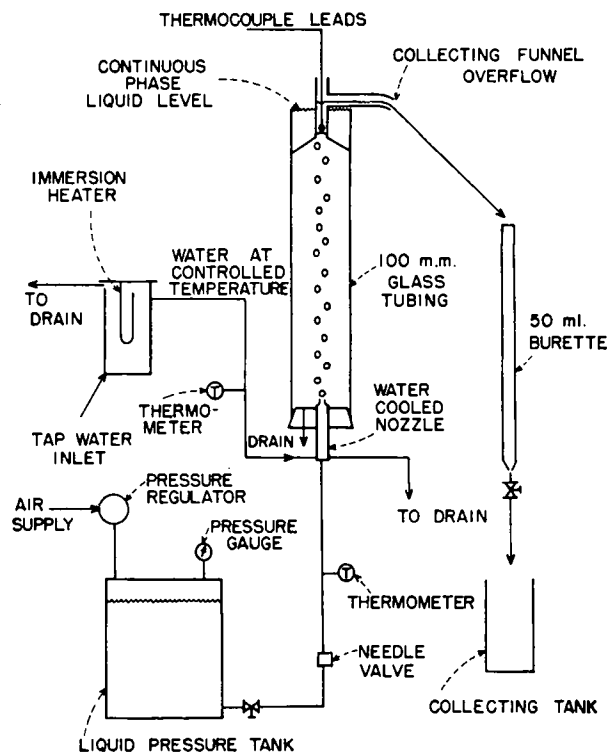


Fig. 3. Diagrammatic sketch of equipment.

these times is obviously not a definite value. However, it can be seen that the faster the drop circulates the more nearly constant the temperature becomes along any streamline.

The equation given by Kronig and Brink for solvent extraction can be written as an expression for the unaccomplished temperature change of a drop as follows:

$$Y_T = 0.653 \exp \left[\frac{-26.84\alpha\theta}{r_m^2} \right] + 0.200 \exp \left[\frac{-157.2\alpha\theta}{r_m^2} \right] + \dots \quad (3)$$

The fact that the type of circulation described by Kronig and Brink actually does occur has been shown in several ways. Spells (16) photographed the striae in glycerine-water drops and his pictures show the circulation pattern clearly. In the studies of mass transfer (6 and 14) the rapid extraction rates measured experimentally were difficult to explain unless circulation within the drop was assumed. Another proof of internal circulation was shown by Heertjes (8) in his studies of mass transfer in an isobutanol-water system. Drops of isobutanol were allowed to rise through water, and the amount of isobutanol transferred to the continuous water phase was measured. In this case the fact that the liquid in the drop circulated was shown by a color change which occurred when the concentration of isobutanol in the drop reached a certain value. Heertjes found that the last remaining portion of the original color had the shape of a ring at position A, as

shown in Figure 2. If no circulation had occurred in the drop, the original color would have been expected to remain longest in the center of the drop.

Hughes and Gilliland (9) have studied the mechanics of drops and discussed much of the theory that has been developed in the past to explain the mechanisms of circulation. Garner (2) has published similar work which includes pictures of drops showing the movements of fluids in and around a drop.

Kronig, Van der Veen, and Ijzerman (13) have considered the transition region in which circulation is slow enough so that the streamlines are not isotherms. For this case the heat transfer rate is faster than for a stagnant drop, but slower than in a drop with more fully developed circulation.

In a circulating drop, new surface is continually being formed at point *B*, as shown in Figure 2. At the same time the surface at point *D* is disappearing. Once circulation has started, the energy re-

quired to form new surface at point *B* is offset by the energy added to the drop by the disappearing surface at *D*. According to Bond and Newton (1), the interfacial tension must be overcome to start the circulation. They developed a relation which defined a critical radius below which no circulation would occur. It was based on experimental data in the streamline region with Reynolds numbers less than unity and is therefore not generally applicable.

Garner and Hammerton (4) have checked Bond and Newton's theory with additional experimental work. The actual radius at which circulation started was usually much smaller than would be predicted by Bond and Newton. Garner and Skelland (5) investigated circulation at higher Reynolds numbers and found that as the interfacial tension increased from 1 to 5 dynes/cm. the Reynolds number corresponding to the critical radius increased from 70 to 120. Garner and Hale (3) have studied the effect of surface-

active agents on circulation and found that small traces of the agents form an adsorbed layer on the drop surface which hinders circulation.

EQUIPMENT

Figure 3 is a diagrammatic sketch of the equipment used in this experiment. The equipment for the formation of drops consisted of a pressure tank, a water-cooled circular nozzle, and an electric heater. The pressure tank was used as a supply storage vessel for the drop-forming liquid. The tank was fitted with a pressure gauge and connected to a pressure-regulated air supply. During runs the pressure in the tank was maintained between 1.5 and 2.0 lb./sq. in. gauge. With this pressure, the desired drop-formation rate could be obtained by adjusting the needle valve.

Three different water-cooled nozzles were used, all constructed as shown in Figure 4. The inside diameters of the nozzles were 2.5, 3.0, and 4.5 mm. A baffle, shown by the dotted line in Figure 4, forced the cooling water to circulate around the incoming stream. The cooling water was maintained at the same temperature as the incoming stream to prevent heating by conduction through the nozzle prior to drop formation. The immersion heater was used to control the temperature of the water flowing around the nozzle.

A rubber stopper was used to seal the column at the bottom. Various lengths of glass tubing were used to vary the time of contact of the drops. The column was filled to the top for each run and the drops were collected and drained off with a special glass funnel, shown in Figure 5. The liquid from this funnel ran into a burette which drained to a collecting tank.

EXPERIMENTAL DATA AND PROCEDURE

Experimental data were obtained for twenty-nine runs and have been reported by McDowell (15). Table 1 shows the physical properties of the systems used and Table 2 the data from one experimental run.

The data required to calculate the unaccomplished temperature change were as follows: (1) inlet-drop-liquid temperature t_i (°F.); (2) outlet drop-liquid temperature t_o (°F.); and (3) continuous-phase liquid temperature t_c (°F.).

The data required to calculate the theoretical temperature rise of the drop were as follows: (1) radius of drop r_m (ft.); (2) time of rise for each drop θ (hr.); (3) velocity of rise u (ft./hr.); (4) physical properties of both liquids.

The inlet-liquid temperature was measured by a thermometer placed in the inlet-liquid stream. The outlet-liquid temperature was measured by a copper-constantan thermocouple placed just above the liquid-liquid interface. The height of this thermocouple was kept as close as possible to the interface between the two liquids. This procedure gave an average temperature of the liquid in the drop just after it broke through the interface. The temperature of the liquid in the column was measured at various heights and radial distances from the axis. At any time during a run the temperatures throughout the continuous liquid phase varied by less than 1°F.

TABLE 1. PHYSICAL PROPERTIES OF LIQUIDS AT 100°F.

	k , B.t.u./(hr.) (ft. °F.)	ρ , lb./ cu.ft.	C_p , B.t.u./ (lb.)(°F.)	α , sq.ft./ hr.	μ , centi- poises
Water	0.363	62.0	1.0	0.00585	0.69
Kerosene	0.084	50.5	0.49	0.00339	1.8
S.A.E.-10 lubricating oil	0.071	56.5	0.45	0.00280	30.0
Xylene	0.090	54.3	0.42	0.00395	0.55
Trichloroethylene	0.080	90.9	0.23	0.00380	0.54
1 part trichloroethylene* 1 part kerosene (by volume)	0.082	70.7	0.32	0.0036	1.2
2 parts trichloroethylene* 1 part kerosene (by volume)	0.081	77.4	0.29	0.0036	0.96
90% xylene* 10% trichloroethylene (by volume)	0.089	58.0	0.39	0.039	0.55

*Physical properties were determined by interpolation between properties of pure substances.

TABLE 2. SAMPLE OF EXPERIMENTAL DATA

Run 14

System: continuous phase-water, drops-oil

Depth of liquid: 44 in.

Rate of drop formation: 105 to 125 drops/min.

Water temp., °F.	Inlet oil temp., °F.	Outlet oil temp., °F.	Time of rise, sec.	Volume of drops, ml.	Unaccomplished temp. change, dimensionless
190	74	129	8.9	0.433	0.525
185	74	129	8.6	0.450	0.505
176	74	127	8.6	0.440	0.480
174	74	123	8.6	0.443	0.510
170	74	119	8.5	0.446	0.531
167	74	118	8.5	0.447	0.527
165	74	117	8.5	0.446	0.528
162	74	116	8.5	0.437	0.523
157	74	116	8.5	0.430	0.495
154	74	116	8.5	0.429	0.475
152	74	113	8.5	0.426	0.500
148	74	110	8.5	0.434	0.514
143	74	105	8.5	0.426	0.550
141	74	104	8.5	0.411	0.552
136	74	102	8.5	0.417	0.548

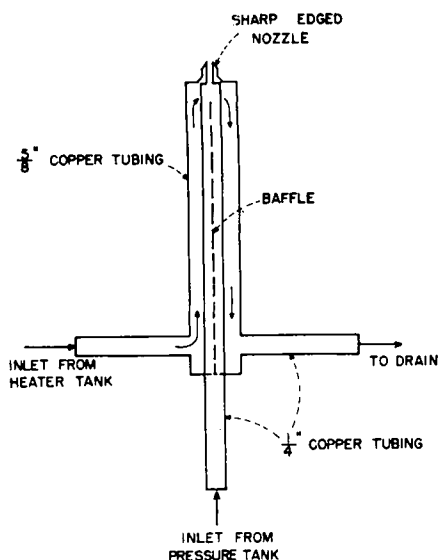


Fig. 4. Water-cooled nozzle.

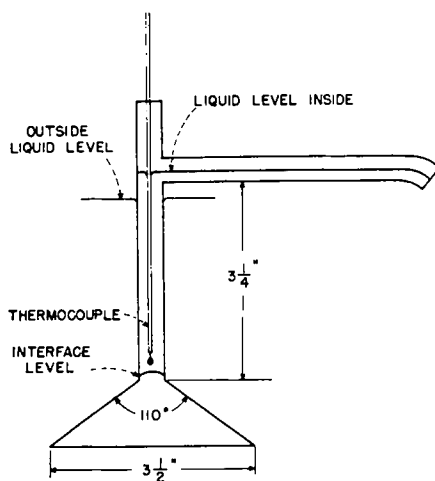


Fig. 5. Collecting funnel.

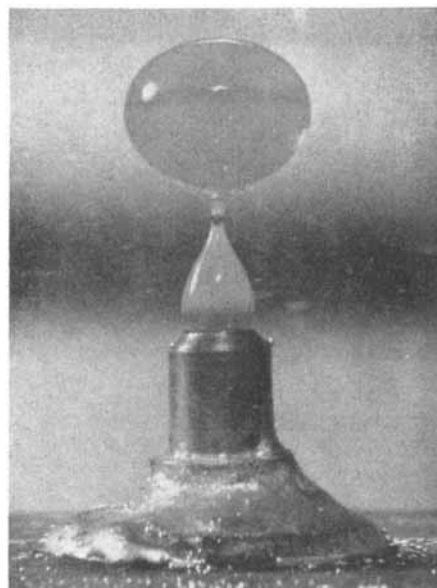


Fig. 6. Oil drop leaving nozzle.

The radius of each drop was calculated from measurements of the drop-formation and volumetric-flow rates. The drop-formation rate was measured in two ways. At slow rates the time required for twenty-five drops to form was measured by a stop watch. At fast rates a Strobotac was used. Drop rates varied from about 100 to 400 drops/min. The volumetric-flow rate was calculated from measurements of the time required to collect 25 ml. of fluid in the burette. From these two measurements the volume of each drop could be calculated. The radius was calculated on the assumption that each drop was a sphere.

The time of rise of the drops was measured directly with a stop watch, the average velocity of rise being calculated from the time of rise and column height.

Figure 6 is a picture of an oil drop just as it is separating from the nozzle. The photograph has been enlarged about four times. Actually the drop is not so flat as it appears in the picture, as the optics involved in photographing through a cylindrical column

of water tend to make the drop appear flattened. If the drop was assumed to be an oblate spheroid with an eccentricity of two, instead of a sphere, the increase in surface area would be only 10%. This fact combined with the experimental results helps to justify the assumption for purposes of heat transfer calculations that the drops behaved as spheres.

RESULTS

If a drop were allowed to remain in contact with the continuous-phase liquid long enough, its temperature would change from some initial value to the temperature of the continuous-phase liquid. The actual average temperature change of the drop may be expressed as the ratio of the unaccomplished temperature change to the total possible temperature change. Thus, if the drop were allowed to heat to the temperature of the bulk fluid this ratio, Y_e , would be zero.

TABLE 3. RESULTS

System	Column height, ft.	Average time of rise, sec.	Drop radius, ft.	Run	Y_e	Y_i		
						I	II	III
K-W	3.67	7.0	0.0113	5	0.15-0.19	0.38	0.39	0.16
K-W	3.67	6.8	0.0128	4	0.21-0.25	0.45	0.46	0.23
K-W	3.67	7.6	0.0144	3	0.24-0.32	0.48	0.49	0.26
K-W	1.17	2.5	0.0118	19	0.42-0.46	0.62	0.64	0.44
K-W	1.17	2.6	0.0138	17	0.48-0.50	0.66	0.68	0.50
K-W	1.17	2.3	0.0140	18	0.50-0.53	0.68	0.70	0.52
K-W	0.71	1.6	0.0111	8	0.46-0.52	0.66	0.68	0.50
K-W	0.71	1.6	0.0128	6	0.51-0.54	0.71	0.73	0.56
K-W	0.71	1.6	0.0143	7	0.56-0.62	0.73	0.75	0.60
O-W	3.67	8.9	0.0146	10	0.49-0.56	0.49	0.50	0.28
O-W	3.67	8.5	0.0154	14	0.48-0.55	0.52	0.53	0.31
O-W	3.67	8.6	0.0177	11	0.61-0.73	0.57	0.59	0.38
O-W	3.67	8.9	0.0181	15	0.56-0.60	0.57	0.59	0.38
O-W	3.67	8.6	0.0186	13	0.58-0.66	0.59	0.61	0.40
O-W	1.17	3.1	0.0178	16	0.75-0.80	0.71	0.73	0.57
O-W	0.71	1.9	0.0144	9	0.71-0.82	0.72	0.74	0.58
O-W	0.71	2.1	0.0178	12	0.85-0.87	0.77	0.80	0.65
X-W	3.67	8.3	0.0156	23	0.15-0.19	0.46	0.48	0.24
X-W	1.17	2.5	0.0117	22	0.28-0.30	0.58	0.60	0.39
X-W	1.17	2.9	0.0143	20	0.30-0.34	0.62	0.64	0.44
X-W	1.17	2.8	0.0158	21	0.40-0.45	0.67	0.69	0.51
X-W	0.71	2.0	0.0124	24	0.45-0.47	0.63	0.65	0.46
X-W	0.71	1.9	0.0146	26	0.47-0.50	0.69	0.71	0.54
X-W	0.71	1.9	0.0152	25	0.53-0.55	0.71	0.73	0.56
0.9X-W	0.71	3.5	0.0199	27	0.48-0.53	0.70	0.72	0.55
W-2,1	2.67	6.2	0.0115	29	0.37-0.44	0.29	0.52	0.08
W-2,1	2.67	6.2	0.0134	28	0.48-0.55	0.37	0.59	0.14
W-1,1	2.67	7.2	0.0143	31	0.50-0.57	0.36	0.59	0.14
W-1,1	2.67	7.4	0.0160	30	0.51-0.57	0.41	0.64	0.19

System

- K-W = kerosene drops in water
- O-W = S.A.E.-10-lubricating-oil drops in water
- X-W = xylene drops in water
- 0.9X-W = 90% xylene-10% trichloroethylene (by volume) drops in water
- W-2, 1 = water drops in solution of two parts trichloroethylene and one part kerosene by volume
- W-1, 1 = water drops in solution of half trichloroethylene and half kerosene by volume

Column height = distance between nozzle and interface in collecting funnel, ft.

Drop radius (r_m) = radius of a sphere of the same volume as the drop, ft.

Y_e = experimental value of unaccomplished temperature-change ratio

Y_i = theoretical value of unaccomplished temperature-change ratio

Italicized numbers in columns II and III indicate the values of Y_i that agree with those of Y_e .

Mechanism

- I = internal radial conduction with no outside film resistance
- II = internal radial conduction plus outside film resistance
- III = internal circulation with no outside film resistance

The smaller the value of Y , the greater the heat transfer for a given drop in a bulk fluid at a given temperature.

In Table 3 these experimental values for each run are tabulated and compared with the values of Y , the calculated theoretical-temperature-difference ratios.

DISCUSSION OF RESULTS

Three different mechanisms of heat transfer were assumed in calculating values of Y , as shown in columns I, II, and III of Table 3. Column I contains the theoretical Y value for drops with radial conduction inside and no outside film resistance. Column II is for drops with internal radial conduction plus an outside film resistance. Column III contains the Y values for drops with internal circulation and no outside film resistance. The Y values were obtained from Equations (2) and (3) and Figure 1.

Internal Radial Conduction with No Outside Film Resistance

When the flow of heat inside a drop is by radial conduction alone, its resistance is higher than if there were circulation within the drop. The numerical values of Y , shown in column I are generally higher than the experimental values, indicating that some mechanism such as circulation must have decreased the resistance of the drops to heat transfer. In the oil-water systems, however, the Y values in column I are close to the experimental values, an indication that there was negligible circulation inside the high-viscosity oil drops.

Internal Radial Conduction Plus Outside Film Resistance

The figures in column II were calculated with the outside film resistance as determined from Equation (1) taken into account. Where water was used as the continuous-phase liquid, this additional factor did not increase the Y values more than 4%. However, the Y values in column II are slightly closer to the experimental Y values for the oil-water runs than are those in column I.

The close check confirms the assumption that heat transfer is by radial conduction internally with an outside film offering a small additional resistance.

Internal Circulation with No Outside Film Resistance

The Y values in column III calculated with this mechanism assumed agree fairly well with the experimental values for the runs in which kerosene, xylene, and organic-liquid drops were heated with water. This agreement indicates that there was circulation within these drops and that any outside resistance was negligible. Circulation would be expected in these cases as the viscosities involved were much lower than in the oil drops.

Internal Circulation with Appreciable Outside Film Resistance

In the last four runs using water drops heated with mixed organic liquids, this was undoubtedly the mechanism involved. Water has a low enough viscosity so that, on the basis of previous runs, water drops would be expected to circulate. As organic liquids were used in the continuous phase, the film resistance would be much higher than for water films. The experimental values of Y were between the Y values in columns II and III. This is to be expected, as any circulation within the drop would lower the Y values shown in column II and any outside film resistance would tend to increase the values shown in column III.

Those values of Y which agree with the experimental values Y are italicized in Table 1, to indicate what appears to be the controlling mechanism. For these cases agreement between experimental and calculated values was fairly satisfactory. The last four runs involved circulating drops with what appeared to be appreciable outside film resistance. As no method for calculating Y for this mechanism was available, comparison of theoretical calculations with experimental results was not possible.

No specific investigation was made of end effects. It might be expected that if end effects were significant they would show up in the form of deviations of experimental results from the theoretical values for the shortest column (0.71 ft.); however, from Table 1 it can be seen that the results for this column are as good as those for the longer columns. Furthermore, it was noted by McDowell (15) that variations of 35% in drop-formation rate produced no change in exit-drop temperature. It is likely that systems involving much higher velocities and shorter liquid columns would be necessary before end effects would contribute appreciably to the heat transfer.

CONCLUSIONS

The following conclusions were drawn from this investigation.

1. If the viscosity of the liquid in the drop is low enough compared with the viscosity of the continuous-phase fluid, the liquid in the drop will circulate. Circulation lowers the resistance to heat transfer.

2. If the thermal conductivity of the continuous-phase liquid surrounding the drop is low compared with that of the liquid in the drop and there is no circulation within the drop, an appreciable part of the resistance to heat transfer will be in the film outside the drop.

3. Theoretical calculations of temperature rise enable the prediction of values that are in good agreement with experimental results.

NOTATION

- C_p = specific heat, B.t.u./lb. (°F.)
 h = heat transfer coefficient, B.t.u./hr. (sq. ft. °F.)
 k = thermal conductivity, B.t.u./hr. (ft. °F.)
 Nu = Nusselt number = $2r_m h/k$, dimensionless
 Pr = Prandtl number = $C_p \mu/k$, dimensionless
 r = spherical radius, ft.
 Re = Reynolds number = $2r_m u \rho/\mu$
 r_m = outside radius of drop or sphere, ft.
 t_c = temperature of continuous-phase liquid in column, °F.
 t_i = temperature of inlet-drop fluid, °F.
 t_o = average temperature of drop as it leaves at the top of the column, °F.
 u = average drop velocity, ft./hr.
 v = volume of drop, ml. or cu. ft.
 Y = experimental ratio of unaccomplished temperature change to total possible temperature change $(t_c - t_o)/(t_c - t_i)$, dimensionless
 Y_t = theoretical temperature difference ratio $(t_c - t_o)/(t_c - t_i)$, dimensionless
 α = thermal diffusivity, sq. ft./hr.
 θ = time, hr.
 μ = viscosity, lb./(ft.)(hr.)
 ρ = density, lb./cu. ft.

Subscripts

- 1 = drops
 2 = continuous-phase liquid

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